The labyrinthology of Pierre Rosenstiehl.

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Rosenstiehl is a mathematician who publishes prolifically in the areas of graph theory, plane convexity and distortion, and computer science (Math Subject Classification areas 05, 06, 51, 52, 68). As the director of the Atelier de Taxiplanie at the Université de Paris IV, he oversees teams of researchers investigating the topology of graph configurations for a variety of applications, such as wiring layouts, printed circuitry, colour mixing and image fidelity, computer graphics, and rigidity in mechanical structures. Software developments arising from this research include automatic schematics, graph partitioning, symmetry detection and realisation, and automatic theorem verification. Collaborative partnerships with industries such as the electrical engineering department of Dassault Aviation and the Laboratoire Cerveau-Cognition take the research into diverse applications.

Four papers drawn from Rosenstiehl's work bearing on the analysis of the cathedral labyrinth layout are discussed here: "Labyrinthologie Mathématique" (1971), "Les Mots de Labyrinthe" (1979), "Le dodécadédale ou l'éloge de l'heuristique" (1982), and "How the 'Path of Jerusalem' in Chartres Separates Birds from Fishes" (1985). Over the course of these papers, he develops procedures for navigating through the labyrinth, at first concentrating on it generically as a curved closed line involving crosspoints, and later charting the specifically unicursal layout as a schematic diagram concerned primarily with the structure underlying the relationship of binary turn sequences. Both "Les Mots" and "La Dodécadédale" expand the analysis to research theory, the former comparing search methods and social structure, the latter, to interdisciplinary cooperation. While "The Path" returns to the technical format of "Labyrinthologie," its use of imagery transcends the limits of the profession as does the context, a symposium on the art and science of M.C. Escher, in a manner anticipated in the two earlier papers. The development in Rosenstiehl's thinking and particularly his use of language in this body of work appears to have been influenced by the structuralist movement of the fifties and sixties, especially the ideas of Roland Barthes, who applied to many other fields that model of linguistics which assumes a deep underlying structure to all human activity and communication.

The Structuralist influence of Roland Barthes

Barthes' translator, Annette Lavers, describes the challenge she faced: "The style reveals a quasi-technical use of certain terms in an effort to account for the phenomenon of mass culture by resorting to new models. Foremost is linguistics, whose mark is seen not so much in the use of specialized vocabulary as in the extension to other fields of words normally reserved for speech or writing."[1] Thus the connotations of the original

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[1] A. Lavers, Translator's Note, in *Mythologies* (London: Collins, 1988), p. 7.

field are brought to the new one by the use of specific words, and this is certainly part of the adventure of following the subtlety in Rosenstiehl's thought. When he speaks of a network (*reseau*), for example, it could be an electronic circuit or equally a group of colleagues; he pivots easily between the naming of parts within mathematical convention, their visual representation, the written word to signify them, and the metaphorical implications of the entire "sign". Even his use of the word "*mots*" and "*concatenation*" rather than "*paroles*" and "*phrase*" or "*sentence*" comes loaded with implications and external references. One senses his desire for dialogue, and indeed in both "Les Mots" and "La Dodécadédale" that issue is addressed. To watch this unfolding from the highly technical field of graph theory, by means of visual and linguistic metaphor, to encompass and indeed formulate a general philosophy of research is to witness an integrative and integrated approach to cross-disciplinary work. That Rosenstiehl does so upon the vehicle of the labyrinth is intriguing.

A brief overview of the related fields of linguistics and semiotics provides some context here. Roland Barthes (1915-1980) was a widely influential French literary critic who developed and systematised the theory of linguistic signs first conceived by Ferdinand de Saussure (1857-1913), the Swiss linguist and founder of modern structuralist (or descriptive) linguistics. Based on an understanding of language as a systematic structure linking thought and sound, Saussure had established that the relationship of the linguistic sign to that which it signifies is arbitrary, and thus a function of cultural convention, not of nature. This means that language is essentially a self-contained system of signs, each element meaningless except in relation to and as differentiated from the other elements in the system.

The American linguist Noam Chomsky's theory, first proposed in 1957, positing innate structures rather than minimal sounds as the basis for speech, established what is known as the transformational - generative school of linguistics, which attempts to define rules that can generate the infinite number of grammatical (well-formed) sentences possible in a language. It starts not from a behaviourist analysis of minimal sounds but from a rationalist assumption that a deep structure underlies a language, and that a similar deep structure underlies all languages. Transformational grammar attempts to identify rules ("transformations," a word of equal significance in tiling theory) that govern relations between parts of a sentence, on the assumption that beneath such aspects of word order a fundamental structure exists.

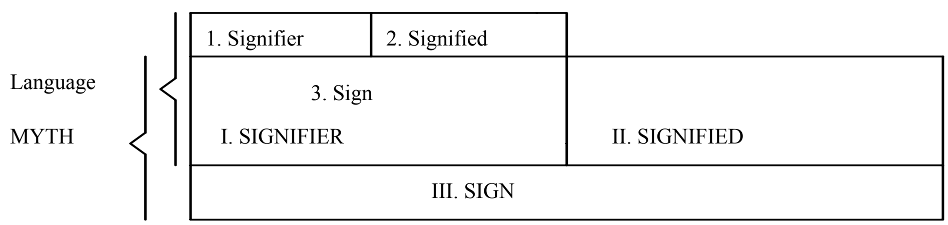
This structuralist model of language has been applied to various areas of cultural study and systems of classification. Claude Levi-Strauss, the Belgian anthropologist, sees human behaviour as a system of communication. By comparing the formal relationships between elements (e.g. mythologies and cosmologies) across cultures, he has found structural similarities in the patterns created by apparently disparate groups. Joseph Campbell has likewise identified a universal theme and pattern, the narrative of the hero's quest, in his exhaustive study of world mythologies.

Roland Barthes expanded on Saussure's original distinction between the *langue*, the state of a language at a given time, and the *parole*, the speech of an individual. In the *langue*, according to Barthes' view of Saussure, the "signified"' is the concept, the "signifier"' is the acoustic image (which is mental) and the relationship between concept and image is the "sign" (the word, for instance), which is a concrete entity (1988, p. 113).

Barthes describes this relation of signifier and signified as being between objects of different categories, and thus one of equivalence and not equality. He further defines all kinds of meaning-laden objects as “words” inasmuch as they carry significance beyond the superficial according to the dynamics of semiology, and are tokens of communication. Thus, in his description of the gift of a bouquet of roses, the roses signify passion, the roses are “passionified”; both the roses and the passion, termed the signifier and the signified, have an existence previous to their current state as the third term, the sign. He differentiates between the roses “experienced” by the observer as a message, and the roses analysed as signifier distinct from sign. The signifier is empty, Barthes says, but the sign is full, it is a meaning.

Word and Myth

Barthes further develops a “second-order semiological system” to encompass what he refers to as “myth.” It is worth looking at this system, illustrated in the accompanying diagram (p. 115), because this is the context that Rosenstiehl refers to when he uses the terms “word,” “language,” and “myth” in both of the Collège du France papers. When Barthes says the tri-dimensional pattern (signifier, signified, sign) operates only to decipher, being about form rather than content and thus actualized in different ways, one can see the outlines in Rosenstiehl's search for the underlying structure of the search itself, as well as of the superficial terrain: the logic of equating Ariadne’s brief words of instruction to Theseus with algorithms, the extrapolation of her Thread to networks of communication, the pattern of the markers of patterns. Most clearly this is suggested in Rosenstiehl’s use of the term “word,” because this derives directly from Saussure, and refers to the “sign,” the third term.



First and second order semiological systems..."the spatialization is... only a metaphor"

In Barthes’ second-order system, the sign, the total of the signifier/signified relationship, becomes a signifier at the second level, that of myth. Because myth uses as material, according to Barthes, all classes of things - photography, ritual, art, gestures, language - inasmuch as these come laden with ascribed cultural meaning, they are used as signifying entities, as “global” signs, they are “language-objects.” Barthes gives the example of the photograph on a magazine cover of a black soldier in a French uniform, saluting, eyes raised, presumably, to the French flag. Barthes says that is the *meaning* of the picture. But what is *signified* is the greatness of the French Empire, that all Frenchmen regardless of colour are proud to serve in her defense, and that France can reject the accusation that she oppresses her colonial subjects. At the mythical, or second-order level, the final sign of the first, language-object system, the black soldier saluting the French flag (= noble, unoppressed colonial) becomes the signifier of the second, and what is signified is a mixture of Frenchness and militariness. So the sign, the final term of the first system, at the level of language, Barthes calls *meaning*. This same term, at the level of myth, becomes *form*. At both levels, the signified is the *concept*. The final term of the mythical level, he calls *signification*, justifying the term by the dual function of myth: “it points out and it notifies, it makes us understand something and it imposes it on us” (pp.116-117).

Having laid out the terminology, Barthes can then describe the dynamic interaction between the two levels: there is much ambiguity. While the meaning of the sign is complete and self-sufficient (the black soldier is an individual with a biography and a country of origin), at a mythic level it becomes empty, the biography of the soldier must be put in parentheses if the picture is to receive the wealth of meaning signified conceptually. Barthes emphasizes here the impoverishment of the meaning by the form: “the meaning loses its value but keeps its life” which the form of the myth draws upon, a reserve of history and riches. The form of the myth “hides” in the meaning: the black soldier saluting is not a symbol, he is too “rich...innocent...indisputable,” but his presence is transparent, and becomes the “accomplice” of the concept of French imperialism, thus artificial. Barthes provides the similar example of the diamond ring; at once a crystalline mineral and potentially a machine tool, the meaning of the stone at a mythic level is stripped and reloaded with the concept of “true gift of eternal love,” to the benefit of the diamond industry, and the great short-circuiting of the complexity of marital relations.

An analysis of the qualities of the myth completes the necessary lexicon (though by no means does any justice to the subtleties of the theory): the third term is the association of the first two, the mythic form and the mythic concept, and both are manifest, not latent. In visual (and presumably ritual and gestural) myth, as opposed to oral, the elements of form are spatial (the layout of the black soldier's picture on the page, the lines of the labyrinth on the floor) while the concept is “memorial...a hazy condensation of a certain knowledge.” The concept distorts the form: the soldier is deprived of biography, the pattern of lines on the floor becomes an initiation. But the form is at the same time the meaning of the first order sign, and Barthes describes this alternation as a turnstile, the point of departure constituted by the arrival of meaning. The process of alternation between meaning and form Barthes compares to sitting in a car looking at the scenery, focusing either on the glass and the distance of the landscape, or the transparency of the window and the depth of the long view, so that the window is at once present and empty, and the landscape unreal and full, like the mythic signifier, whose form is empty and present, and sign/meaning unreal and full. To stop the spinning of the turnstile of form and meaning, Barthes says one must “apply to myth a static method of deciphering...go against its own dynamics.” Analysis of Rosenstiehl's work on the labyrinth demonstrates that he has undertaken to do just that.

The labyrinth of mathematics

Labyrinthologie Mathématique is a dense technical paper concerning the implementation of the algorithmic theorems of Tarry, Hamilton, Trémaux and Euler when transversing graph planes, depending on whether or not one wants to cross the same point more than once, if at all. Analogs of this situation are “the taxi round Paris, visiting all the gates,” and “getting the postman back to the post office.” An example of the application of this problem is found in parallel process programming in computer engineering when setting up nodes for searches, whether “depth-first” or “breadth-first.” Each node has valences of pathways connected to it, which may or may not lead to the goal node. The difference in search method can be visualised as the circumnavigating of a polygon (breadth-first: all the gates of the city) or going down each of the branches of a tree (depth-first: the postman). The relevance of this distinction will be evident to anyone who has surfaced after an eternity on a search engine in the Web.

Rosenstiehl states at the outset the two imperatives of a formal systematic "assault" on the mathematical labyrinth: economic exploration, and limited means, achievable through judicious use of graph algorithms, general research procedures, and a structuring of data. His intention is to identify a set of finite automata that could be used at the crosspoints, without regard to the dimensions of the space, such that a circuit could be created of the shortest route between the points without cycling back on any single line between points, and addressing the restriction of crossing or not crossing at the junctions. These automata relate only to themselves and the neighboring nodes of the network: Rosenstiehl compares them to biological, sociological, and technological networks which have the appearance of organic coherence when they are in fact regional developments.[2]

He sees the dual aspect of the solution to the labyrinth: on the one hand, to find a route through the maze of routes such that one can retrace one's steps,[3] and on the other, to define the parameters of automata that can construct such a route, usable both for a single mechanism "in the centralised mode habitual to classic algorithms" and also to those dispersed throughout any labyrinth to be solved.

His approach is to treat the whole problem as a tree, using the image of Ariadne's Thread, and the cyclings back from the tips of the branches he identifies with letters such that each return ("*replies*") to a node is identified using the letter and its inverse indicated as prime: *a a'*, *b b'*. The collection of the words thus formed can be thought of as parenthesised for the purpose of both simplification and for localizing an area of operations. Thus in a sequence, or word, when two letters are not contiguous, the letters in between signal a departure along one or more paths, and this "isthmus" is always characterised by the doubling of each letter within the parenthesis. Using this system, decisions regarding procedures at crosspoints and junctures can be made.

The article, written for mathematicians, goes on at length and in depth on the implications for a variety of problems both classical and more recent, by comparing outcomes such as erasures of isthmuses (*mots vides*), crossings, knots, and closed loops that arise from applying the theoretical approaches mentioned earlier. The idea germane to the analysis of the unicursal Chartres labyrinth is sufficiently outlined here.

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[2] This reference to global and local operations is a technical term specific to plane division, tiling, and tesseration in Euclidean, hyperbolic, and spherical topology, relating to transformative properties of the various groups of operations.

[3] Described as technically a *flèche* (arrow, spire) of a groupoïd, a "word" in the "language" of Dyck.

Labyrinth as a metaphor for interdisciplinary research

The next two articles, "Les Mots du Labyrinthe"[4] and "La Dodécadédale," were presented at the Roland Barthes seminars on The Labyrinth as Metaphor for Interdisciplinary Research at the Collège du France, and have consequently a broader range with reference to the critical structuralist context, as one would expect. In "Les Mots," Rosenstiehl translates the working lexicon of the mathematical labyrinth into the metaphorical language of research, describing the labyrinth "of which each reconstructs a piece of the map by their own efforts" as being "for the artisans of science the archetype of the field of research, and the beating of the bushes, the spirit of research." He offers four observations on the labyrinth:

- the labyrinth is extricable (and he could help shorten it)
- the labyrinth is a grammar, and Ariadne had parenthesised there
- the labyrinth is a myopic algorithm
- the labyrinth is an "a-centred" universe.

Rosenstiehl's primary concern is to develop a theory for maze threading (he uses the word *battre* throughout, meaning the beating of bushes to flush gamebirds) from which may be derived rules and procedures - that is the role of theory. He is condescending towards "architects and mystics" who insist on bumbling blindly towards the center. Strategy is essential in any labyrinth, and while it is human nature to create labyrinths both mental and concrete, mythology provides us with strategies to overcome our "myopia" as well as our monsters. Even the master-builder Daedelus had to resort to tricks to escape his own creation. Rosenstiehl describes the algorithmic theorems of Tarry (1895) and Trémaux (1882) as seminal, "highly distilled, in the language of the period," but obscure: a combinatory artifact by which to escape, but awaiting the theoretical concepts developed more recently in the field of algebra, in order to become accessible as tools. Algebra reveals structures, including that of Theseus' wandering through the labyrinth. This has been transcribed into a formal language, "*l'Algèbre des grammaires génératives*," with notations for word forms and syntax, and thus a subject of interest for the structural theorists at the Barthes seminars. He notes his own response to the labyrinthine set-up of the seminar, both the site and all the presentations: "I have followed, and sometimes I have not followed... the Daedelus of the seminar, ...the signs sending one to the others, ...my desire to deconstruct...to abstract," pointing out the distinction between critical and mathematical language in the use of metaphor. Barthes is quoted: "the theory continually recuperates," and so Rosenstiehl will "recuperate" the word labyrinth, but only for the purpose of the presentation, as it is left by the wayside in mathematical terminology.

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[4] An earlier version of "Les Mots" was presented at the *Colloque sur la Theorie des Graphes* in Brussels in 1973, and published in the *Cahiers Centre Études Recherches Opér*. 15 (1973), pp.245–252. It concerns the properties of "walks on a graph," described by R.C. Read in the Math Review as "a word each letter of which denotes an edge in one or another of its possible orientations."

To lead the audience through the concept of "extricability," Rosenstiehl introduces a schematic view of space: a labyrinth is made up of corridors incident at both ends to a crossroad. A crossroad incident to a single corridor is a cul-de-sac. A corridor incident by both ends to a single crossroad is a loop. A large column in a hall constitutes a loop incident to a crossroad-hall, by which the traveller tours the column. Subways, escalators, moving sidewalks are corridors. The form of crossroads incident to corridors, points and lines, is a graph, or network. But all graphs are not labyrinths; rather, it is the degree of what Rosenstiehl calls "myopia" in the traveller, the limiting of calculations to the local operation, that makes a labyrinth.

One needs to be familiar with two extreme graph forms: the polygon, equal numbers of points and lines alternating (the gates and boundary boulevards of Paris), and the tree, more lines than points, the presence of culs-de-sacs, no polygons. The "beating" of either sort involves the rule that in any crossroad except for the first by which one initiated the journey, the corridor of discovery of that crossroad is not taken except as a last resort. In this way, economy of means is attained, because one only goes once in each direction on any corridor. Rosenstiehl then says that any graph, or labyrinth, is a combination of the two, and that supplementary lines connect the tips of the tree, the culs-de sac, forming polygons, thus constituting a connected finite labyrinth. It was the discovery of Tarry that the labyrinth limits itself by the same rule of not using the corridor of discovery except on final retreat, that is, that the point of arrival is the same as the point of departure. Rosenstiehl dismisses the "always turn left" or "always keep one's right hand on the wall" techniques as "scanty geometric props" suitable only for extremely limited situations and completely useless in the abstract labyrinth of information systems, both social and technological. In support he offers a cautionary reference to Borges' *Library of Babel*, and later, in *The Path*, an amusing drawing of the truncated schematic of the unicursal cathedral labyrinth using such a method. He further illustrates Tarry's rule ("the minimal order, necessary and sufficient") by opening a sequence of parentheses on order and liberty, exploration in each era of newly accessible parts of the labyrinths of mathematics, what this means for creativity, the activity of the cubists all painting the same subject with such varied results, the narrow-mindedness of imposed academic method, and the interchangeability of " (a)maze" and "astonishment" and "maze" and "labyrinth" in English, before neatly closing all the digressions, making his point to the structuralists, and segueing gracefully to the subject of grammar.

Algebraic Grammar

Rosenstiehl introduces the algebraic grammar developed by Schützenberg and Chomsky.[5] It is impossible to say whose was the greater debt in this exchange. The familiar letters are strung together to form words: a passage down a corridor is labelled *a*, the inverse

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[5] Rosenstiehl notes that this is technically known as a monoid, an algebraic structure consisting of a set together with an associative binary operation, i.e. addition and multiplication, for which there exists a left and right identity element, called the unit of the monoid. The example given is: the set of integers considered only with regard to the binary operation of addition, has unit 0. (Integers, addition, 0) forms a monoid, as does (real numbers, multiplication, 1). Frequently used in algebraic geometry, the associative quality can be manipulated to eliminate repeated operations and simplify topographical notation.

direction *a'*. The "prime" operator is the return, and *a a'* = *a' a*. (*a a' a, a, a a' a a'* is groping about blindly in the same corridor). Three corridors in a row and the return, *a j t t' j' a'*, is called a concatenation, which one may do freely, or subject to the rules of grammar which accepts some words and constructions and rejects others. The Chomskyan concept of "context free" grammar is illustrated by the example of a string of one-directional corridors α concatenating with another β to form the word $\alpha\,\beta$, but only if the last letter of α accords with the grammar governing its concatenation with the first letter of β. The rest of the letters are context and of no concern: "depending on where you come from, you can continue. And myopia is permitted."

Concatenation can be seen as parenthesising, like smaller branches on a larger limb, the distance between a letter incongruent with its prime indicating the complexity of routing. Here Rosenstiehl illustrates with Theseus arriving back at the entry point to return his end of the string to Ariadne: when she pulls them together the labyrinth is "annulled," the string, unknotted, parenthesises Theseus. The critical choice is whether to close the last parenthesis opened or carry on and open a new one. Here Rosenstiehl distinguishes between Foolish and Wise Ariadne as search techniques. The Foolish goes on opening parentheses, until there are none left, then rewinds the string, retraces the steps, and closes the parentheses in sequence. This is the most economical of directives, one of the two primary rules stated at the outset in *Labyrinthologie*. The Wise would rewind as one proceeds, that is, closes the last one opened, does not enter a crossroad previously discovered, but otherwise explores new corridors. Both adhere to Tarry's avoidance of the corridor of discovery, and fulfil the requirements of parenthesage. Depending on the nature of the search, each has advantages and disadvantages to the characteristics of their particular translation of the labyrinth structure.

Myopia

In the section on myopic algorithm, Rosenstiehl discusses at length the nature of the signs "posted" at the crossroads. A subject first introduced in *Labyrinthologie*, where the information required of the automata included the shortest path between two points, maximal arboricity, and the existence of circuits, eulerian and hamiltonian cycles, here the distribution of signs among "beatings" according to the rules of Tarry, Foolish, and Wise Ariadne are illustrated and compared. Signs indicate whether a corridor has been explored (the absence of a sign is a sign) and if it is the corridor of discovery and thus to be avoided until final departure. Tarry's rule requires both of these signs, and the two Ariadnes are a class of Tarry's, though Foolish Ariadne requires the further signing to indicate from which corridor to which corridor her Thread runs at a given crossroad. The signs are only "legible" at the ends of the corridors, thus constituting "myopia"— one cannot see what is posted at the other end of the corridor. If an algorithm is a procedure which directs a calculation and produces a result, then a myopic algorithm is one which operates identically all over a field but is aware only of the information available locally, exactly like a traveller in a labyrinth. Thus, according to Rosenstiehl, Tarry's rule is a myopic algorithm.

As most algorithms for path-threading on a graph have general information available to them (the big picture), most are not myopic, and do not belong on labyrinths, but most graphs are not labyrinths. An example of a myopic algorithm successful in a nonlabyrinth situation, however, is that of the solution Rosenstiehl proposes to the famous March of the Knight on the Chessboard problem, where each square must be visited but once: *place the knight each time in a square where he dominates the smallest number of squares not already visited*. He points out that the myopia here extends to two corridors, and that the knight himself is a little myopic.

The Hapax

Rosenstiehl concludes "Les Mots" by generalising the concept of myopia to technological and social information networks, the "acentric universe." Though the dimensions of the labyrinth can be defined as the number of crossroads, and the complexity as the number of signs,[6] nevertheless the rule is universally applied according to local intelligence available, independent of dimension. Each crossroad is a finite automata posting a statement concerning its condition, and communicating this statement along the corridors to its neighbours on a regular basis, until such time as the conditions no longer change and the calculation, now completed, is terminated with a final statement which is the record of the "beating," or search.

This final statement, Rosenstiehl suggests, is a *hapax*[7] (a technical term in linguistics: *hapax legomenon*, plural *legomena*) a word spoken only once. He gives an example of a possible collective hapax: we all show up next Saturday with blue lips, the sign for this would be unique, used only once. But his planning of the event has transgressed the definition, which includes the condition of a-centric. All the friends (the automata) in the network habitually talk (exchange signs) once a week, not keeping track of the number of conversations (signs) or the number of friends (automata) as this would be beyond their capacity. The sign reserved for the hapax is posted in unison by each and every one of the friends (junctions, automata) as the final statement of condition (We're all showing up Saturday with blue lips!). The theorem of the collective hapax sounds like a Hundredth Monkey scenario, a change of state when critical mass is reached in social organisations:

> Theorem of the collective hapax: *An automata exists for new signs by the junction, of which any number of copies (as large as one wants), arranged by their junctions in any connected labyrinth, are able to effect, upon the initiative of any one among them, a collective hapax.*

The agreements of bees

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[6]Similar to Fisher's "span-square: an imaginary square measured along the length of a path, based on the maze's span unit (e.g. centre-line to centre-line in a pavement labyrinth). The number of span-squares containing nodes can be expressed as a percentage of the total number of span-squares in a maze. This is one useful way to evaluate the difficulty of a maze." Fisher, p. 9.

[7]From 'απλου (Greek): simple, single. No Latin transitional word found to the possibly related *happer* (Old French): to snatch, seize, with the beak (v. tr). The English words based on *hap* (happy, hapless &c.) originate in Old Norse *happ*: good luck, and Middle English *happenen*: to occur (v. intr.). The lips referred to further on would be blue from pecking, possibly.

Rosenstiehl compares this phenomenon to the collective agreement of bees to swarm and concludes that the centre of decision is not located in the spatial, in the hexagons of wax evoked by Borges' description of the labyrinth-library "sphere of which the true centre is some sort of hexagon, and of which the circumference is inaccessible," but rather in bee-communication itself. Acentrism is characteristic of "ant-hill" societies, and the network of finite automata in human communication systems, whether biological or technological, is the spontaneous emergence of this, in spite of the historical imposition of hierarchy upon language, concept, and social organisation. He closes by emphasising that the abstraction of the labyrinth gives hope to the emergence of the acentred as a language.

Several levels of interpretation are possible here: a strictly technical application of theory to the problem of the labyrinth as architectural site, an algorithm for the search thread in the computerized graph environment, a schematic of the larger abstract environment of communication technology networks, and ultimately, a metaphor for social organisation, whether beehive or interdisciplinary research seminar. Following the basic tenet of structuralist theory, Rosenstiehl searches for the structure underlying the patterns revealed at each level of analysis, keeping in mind, as he focuses like a microscope at different magnifications, that the organism remains the same no matter how different it may appear. Given his commitment to interdisciplinary work, it is fascinating to read the lists and abstracts of the projects on which he has collaborated, particularly in the areas of neural networks and visual perception. Is there an internal hapax of synapses? An algorithm of political will ? A mathematics of collective action ? Scanning the index of *La revue Mathématiques, Informatiques et Sciences Humains*, the publication of the Centre d'Analyse et de Mathématiques Sociales which Rosenstiehl directs, one would think that if there is, these people will find it.

The *Dodécadédale: Eulogy for the heuristic*

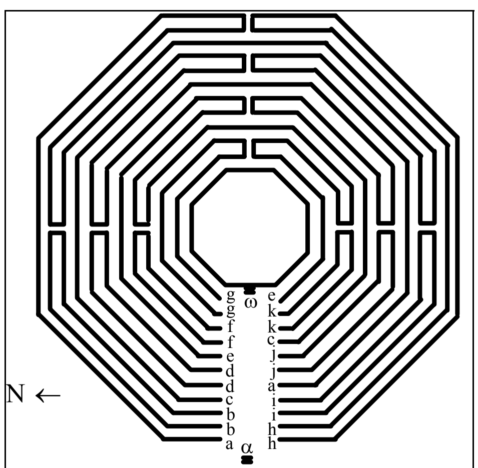
It is with anticipation that one turns to *Dodécadédale*, the second of the Barthes seminar papers. The emphasis here is very different, on two counts. The first is that Barthes had died the previous year, and the tone of the paper is mournful, elegiac, quite unlike the joking and camaraderie of "Les Mots." The second factor is that the technical content of the paper is completely concerned with the unicursal pavement labyrinth, so the now familiar tools of parenthesage are brought to bear on a specific architectural array. Having developed the theory in the abstract and general, the procedure is now used on the particular with significant results. The combination of factors, however, gives the impression of two different papers cobbled together: although the introduction to the historical and experienced labyrinth employs the techniques and vocabulary of structuralist analysis, the closing section, on Barthes' heuristics and the essay *The Winter Garden Photograph*, seems to be an addendum. Nevertheless, the developments in both structuralist and mathematical analysis of the labyrinth are substantial, as Rosenstiehl addresses the metaphor of the "concrete" sign.

The essay opens with a description of the octagonal black and white pavement construction in the parish church of St. Quentin. Right away the language locates the action in the world of experience, not theory: "the black motif unravels like wool"... "traps itself within multiple jaws it has drawn"... "escapes as if by a miracle." "Enormous, enigmatic," it is also threatening: "epileptic vibrations...agitate the senses," "it conceals illusions...baffles the eye...impossible to verify that...the magic trail...is not a trap.” To obtain proof, there is nothing to be done but “throw oneself bodily into the ...unpredictable volutes” and even then it remains questionable whether one has covered the entire course—experience alone does not provide an infallible map of the “Path to Jerusalem.”

The historical background provides little explanation. The builders left few records of either their intentions or their calculations, and it is unknown how much of this may have been suppressed. Both communal games and solemn rituals of reflection, pilgrimage and penance are said to have taken place upon them. But in the absence of received evidence, all one has to go on is the fact of the pattern on the floor: “what one may say of it appears simply to be about the form itself...the topical is richer than the general, the denoted richer than the connoted, the literal richer than the symbolic.” It is not a symbol: unlike the cross, it has no dogma attached, it is “multivocal,” a cue for anyone’s private unfolding. Nor is it a myth in the structuralist sense of collective agreement on interpretation: the labyrinth serves everyone as “the geometry of welcome” for private, spiritual, site-specific events. Nor is it metaphor: it is illogical, there is no connection between the visible signifier-pattern and the signified journey of the soul. Not any of these, but rather, says Rosenstiehl, the labyrinth is “a method for the arrangement of a multitude of metaphors.”

The law of alternation & the law of doubling

But there is geometry! By locating the irregularities, one can establish a “law of alternation” regulating the sequence of straight runs and opposing turnabouts at the ends of the three semi-axes at the North, East, and South sides of the pattern. The throat, on the West semi-axis where the two ends of the actual path come out in the centre and the entrance, is where the labyrinth “plays.” Including these paths of access to centre and exterior as elements, the strands are matched and form turnabouts away from the axis, in a system of paired turnabouts nested within a larger single span, two layers of nests on each side creating the four tiers of the throat, evidence of the “law of doubling.” The same form is to be found consistently in all the examples documented or still in existence. How can this be?



St. Quentin labyrinth path, threads of the throat matched and labelled (prime not indicated).

Rosenstiehl uses the concatenation, or parenthesage, method, to demonstrate that the labyrinth layout is the only solution to a system of equations, given the laws of alternation and doubling, and he believes that this was a fact known to the medieval builders. Starting at the exterior of the East semi-axis, one encounters a turnabout, and then, moving towards the centre, an alternation of straight lines and turnabouts. Should one try, at either the North or South semi-axes, to start with a turnabout rather than the existing straight runs, a closed loop would result with the outer turnabout in the East. The arrangements at the three semi-axes are therefore givens, and the only options remain at the throat.

Here is Rosenstiehl verbatim on the application of a familiar technique:

Italic letters are assigned in the throat to the eleven strands from the NORTH and the eleven from the SOUTH, α to the exterior access, ω to the central passage, and identical letters to designate the two ends of the same thread determined by the decisions taken on the three other semi-axes. One thus obtains the two series of threads of the throat:

$\alpha\ a\ b\ b\ c\ d\ d\ e\ f\ f\ g\ g$ for the threads from the NORTH,

$h\ h\ i\ i\ a\ j\ j\ c\ k\ k\ e\ \omega$ for the threads from the SOUTH.

To resolve the equation requires parenthesising the above two words such that:

i) matching the letters two by two in the parenthesage creates a single thread running from α to ω;
ii) the law of doubling is respected, which signifies that any parenthesis belongs to a complete group of parentheses of height two exactly, which assigns us to one of the five following schemas:

(.(..) (..) (..).) (.(..).) or (.(..) (..).) (.(..) (..).) or (.(..).) (.(..).) (.(..).)

or

(.(..).) (.(..) (..) (..).) or (.(..) (..) (..) (..) (..).)

A systematic analysis reveals finally that for the above words a single parenthesage-solution complies with conditions (i) and (ii). It concerns the following parenthesage:

$$(\alpha\ (a\ b)\ (b\ c)\ d)\ (d\ (e\ f)\ (f\ g)\ g)$$

$$(h\ (h\ i)\ (i\ a)\ j)\ (j\ (c\ k)\ (k\ e)\ \omega)$$

which represents none other, in fact, than the solution of Saint-Quentin. With a little bit more courage one verifies that the alternatives left by the wayside on the three

> semi-axes do not support any other solution. *A priori,* 675[8] schemas were possible, but only one provides a path that does not cycle back on itself. We will call this the *Dodécadédale*[9]

Rosenstiehl goes on to examine a "labyrinth of Solomon" in a sixteenth century Venetian manuscript with a somewhat different arrangement on *d* and *e*, which does not adhere to the law of doubling, and is "badly drawn." He speculates on the possibility of a mathematical law of perfect parenthesage, emphasising its utility in establishing the laws of alternation and doubling that characterise the "perfect maze in twelve layers."

Barthes' heuristic prefix

As to the mystery of the labyrinth, the reticence of the builders, the transmitted yet unarticulated message of deep devotion to "canonical entanglement," he throws up his hands and leaves it to the historians, before launching into a generalisation on the labyrinthine nature of the thread of research. There he identifies Barthes' use of the phrase "one can say that..." as a "heuristic prefix" which in contrast to the more scientific "it must..." and "it is therefore necessary..." invites exploration on unknown terrain. The image he provides is the grappling hook in Andrei Tarovski's *The Stalker*, thrown each time in a new direction across levels of forbidden territory. His language is vivid and active: "geometry isolates the pure configuration," "throws challenges": "erratic problems spring up" (gush forth, fly as sparks*: jaillisent!*). Barthes' purpose is the framing of the research question itself, the origin of his *angoisse stochastique*. The thread can give form to the search but not authority. The conjectural "it can be said that..." not only invites discourse but also composes the working formula for the underlying structures, to be found in neighbouring fields given the equivalent vocabulary translations and sometimes generative transforms of the idea entirely, as the Chomsky-Schültzenberg collaboration illustrates. (Similar interdisciplinary conceptual transforms can be seen in the field of bio-engineering where the "haptic community"[10]is currently developing applications in psychophysics of human-computer interfaces, Virtual Reality environments for tactile sensing and distance manipulation on a pantographic principle of an activity such as medical surgery, at the neural network level.) The challenge is irresistible: Barthes asks the question and throws out the ball of wool, engaging in the quest the individual and the network.

Experiential grammar

There is an intriguing situation that should be mentioned, before leaving *Dodécadédale*. Investigation reveals that the parenthesage of the cathedral labyrinth, if prime were indicated as in "Les Mots," would result in a string of words clearly articulating (the term choreographing comes to mind) the presence of three identical constructions arranged in

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[8]The figure of 675 given for potential alternatives can be factored out to 3^3 x 5^{2}, which represents the three axes and the five sets of pairs on each side remaining if one eliminates the $\alpha\ \omega$ pair for which there is no variation possible.

[9] Dodecamaze.

[10] http://haptic.mech.nwu.edu

the inner, middle, and outermost layers of the labyrinth. These sequences, on *e*, *c*, and *a* respectively are obvious in the labelled circuits described in “Le Dodécadédale” (see Figure 3).

The problems associated with labelling prime in the drawing are also revealed. Because the labyrinth is unicursal, there really are no choices to be made or crossroads requiring them. Elsewhere, in “Les Mots,” the labyrinth at Knossos is characterised as a cul-de-sac with two crossroads, the entrance and the centre, incident to a single line.

Rosenstiehl has labelled the threads myopically, in relation to the “local information” necessary and sufficient to note incidence to the region of the throat, but not the activity of entering and exiting. As the arrangement of runs and turnabouts is predetermined on the North, East, and South semi-axes, then what one is looking for in the throat is the distribution of turnabouts such that looping will be avoided. Like the knight on the chessboard, distance vision is limited, in this case to the first axis out from the throat, except for *g g′* and *h h′* (which also share the characteristic of being the most extremely situated, in the centre and the exterior circuits 1 and 11). Thus we can see that *a*, *c*, and *e* are distinct from the other threads in having their other end, or prime, in the opposite side of the throat axis from their point of origin.

Picking up the Thread and labelling prime (as a function of location rather than state of discovery or exploration) on *a*, *c*, and *e* would demonstrate some interesting occurrences of symmetry. Table 1 clarifies this issue.

Table 1. Step Sequence Notation

Step #	1	2	3	4	5	6	7	8	9					
Turn			L 90	R	L 90	L	L	R	L					
Quadrant		1	1	1	1	1, 2	2, 1	1	1					
Thread	α	T1	d	d '	T2	g	g '	f	f '					
Circuit		11-7	7	6	6-1	1	2	3	4					
Step #	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Turn	R	R	L	R	L	L	R	L	R	L	R	L	R	L
Quadrant	1, 2	2	2, 3	3	3, 4	4	4	4, 3	3	3, 2	2	2, 1	1	1
Thread	e	e	e	e	e '	k	k '	c	c	c	c	c '	b	b '
Circuit	5	4	3	2	1	2	3	4	5	6	7	8	9	10
Step #	24	25	26	27	28	29	30	31	32	33	34	35	36	37
Turn	R	R	L	R	L	L	R	L	R	R 90	R	L	R 90	
Quadrant	1, 2	2	2, 3	3	3, 4	4	4	4, 3	3, 4	4	4	4	4	
Thread	a	a	a	a	a '	i	i '	h	h '	T3	j	j '	T4	ω
Circuit	11	10	9	8	7	8	9	10	11	11-6	6	5	5–ω	

The table is read in sequence from the viewpoint of entering the labyrinth. It indicates:

- the direction of the turn that begins each step

• the quadrant(s) where the movement takes place, numbered one through four clockwise from the lower left
so that the direction of movement across quadrants is noted
• the letter labelling the segment(s) of the thread or path
• the concentric circuit the movement occupies, numbered one through eleven from the centre outwards.
• throat segments are labelled T1 through 4 consecutively as they would be encountered on the Thread. T1 is the outer left, T2 the inner left, T3 the outer right, T4 the final avenue to the centre ω.

Thus it can be seen that *e e′*, *c c′*, and *a a′* are each five steps, of which three cover two quadrants and the intervening steps only one. Each word moves through all four quadrants and five layers of circuits in the "enunciation." They are separated from one another by one "empty" word in each case. The words *e e′* and *a a′* begin in the first quadrant and end up in the fourth, unlike *c c′*, but apart from the direction of the originating turn, and the fact that, while the overall movement of the former two is clockwise, that of *c c′* is counter clockwise, all follow a sequence of right/ left/ right/ left. That is to say, in each case, all aspects of their movement and location in the labyrinth follow the law of alternation established earlier.

It is worth recalling that the experience of walking these circuits of varying circumference involves a complex series of decisions about movements and gestures. The process of "throwing oneself bodily" into the journey is, as Rosenstiehl points out, more visceral than visual. The number of footsteps required before the sudden halt and negotiation of a turn to the left or to the right is different for each segment, the handedness (footedness rather) of the agent is a factor, as is the choice of foot to lead with when initiating the turn, from a choreographic point of view. It is possible that the repetition of these three elements generates some of the meditative, soothing, or trance-like states experienced when walking the labyrinth: as with repeat movements in dance or T'ai Chi, the mind becomes conditioned to the sequence, and sub-consciously recognizes it when it reappears. The dancer's research question would ask about the relationship between the lengths of the arcs of the concentric circuits in a given "word," how many steps difference there is between *e e′*, *c c′*, *a a′*. What is the underlying rhythm of this movement through space? These are questions which arise from the characteristics of the pavement labyrinth as architectural site-dance floor. But Rosenstiehl leaves behind the Word, and moves on to the pattern at an entirely different plane of magnification.

The Escher Congress

"How the 'Path of Jerusalem' in Chartres Separates Birds From Fishes" was presented to an interdisciplinary Congress on the artist M.C. Escher held at the University of Rome in 1985. Organised by the Mathematical, Physical, and Natural Science Faculty, the sections of the congress were divided into Symmetry, Mathematics and Visual Perception, Geometry (where Rosenstiehl presented), Cinema and Computer Graphics, the Physical World, Art, and the Humanities. The publication of the Proceedings is encyclopaedic in its range, providing verifications of earliest possible dates of historical influences[11]and careful descriptive analyses of comparative classification systems and application procedures to document how Escher had gone about developing his own working system of elements entirely independently and, it would appear, in advance of the academic crystallography field. Rosenstiehl's contribution stands out. Informed by the structuralist practice of searching for the underlying framework in the presenting situation, he uses an austerity of means and a distinctly pragmatic approach to make a conceptual leap of breathtaking inventiveness from the earlier theory of spatial analysis.

Taking an action immediately suggestive of the "acentred" universe of "Les Mots," he dispenses with the rule of doubling (recall the absence of sign for prime, noted in "Le Dodécadédale") and, restricted only by the alternation rule, he opens the cathedral labyrinth at the axis of the throat, "bending" and "morphing" the array into a rectangle, in essence liberating the axes of the labyrinth from their layout radiant from the centre (see figures 4 and 5), while keeping intact the sequence of turns and the relative positions of the opposing turnabouts. Using a visual form of the labyrinth based on the three colour marble labyrinth at the church of Santa Maria in Aquino in Rome, Rosenstiehl maintains the separate identities of the left hand and right hand walls and the path traced between them. Lifting the path out, and cutting off what he perceives to be the "frame," including the throat sections, results in a piece that one can see is part of an infinite array of repeated tiles in two colours. Rosenstiehl concludes his visual presentation by gradually distorting this repeat pattern into a tiling of birds going one way and fish going the other, an "Escherization." Thus, he says, does the path of Jerusalem separate the birds from the fishes, "the air from the water." In one stroke, he has engaged language, history, geometry ancient and algebraic, medieval and late twentieth century cosmologies. The turnstile of form and meaning is a blur.

Labyrinthine Isometries

The technical specifications of the transformed labyrinth are detailed extensively. The repeat pattern is based on an elemental unit called a hexomino, a horizontal row of five white squares with a sixth placed on top of the second square. The counterpart, originally grey in the Santa Maria version, is formed by the operation of rotation on the middle top edge of the fourth square. This creates the fundamental region. The text says of the white and its grey counterpart "both generate the array by repeating the two translations (6, 0) and (-2, 2) and their opposites." Some technical context clarifies this.

Translation and rotation are terms for two symmetry operations that, performed on the motif, or fundamental region, result in its repetition over the whole plane. This transformation relating two congruent figures is called an isometry; translation and rotation are direct isometries. (Reflection and glide reflection are called opposite

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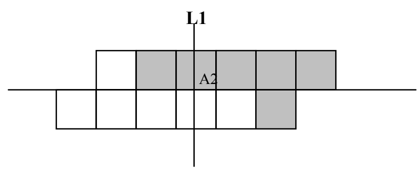
[11] Escher first read Pólya's article of 1924, on the seventeen crystallographic groups illustrated as symmetry groups of tilings, in 1935, according to Branko Grünbaum, in: *M.C..Escher: Art and Science*, Amsterdam: Elsevier, p. 54. H.S.M. Coxeter, the illustrious University of Toronto geometer, introduced Escher to divisions of the hyperbolic plane in 1957. Escher appreciated the illustrations but claimed the "hocus-pocus text" of no use to him! Quoted by Doris Schattschneider in the same volume, p. 82.

isometries because they mirror across an axis.) Symmetry operations include translations in two directions, according to restrictions defined by crystallographic classification: admissible periods for a rotation are 2, 3, 4, and 6. A rotation of 180° or half turn, is called two-fold; 90° or quarter turn is called four fold; location of point of rotation (e.g. midpoints and vertices) further classify types of tilings. The hexomino "escherized" falls into Escher's BIII classification (polygons, two-fold rotation, reverse colour; translation in both diagonal directions; BII classification in that rotation is on two axes rather than four, at midpoints of opposite sides) and the crystallographic system of P2. Shared edges of such tiles are opposite colours, tiles sharing vertices are same coloured (Schattshneider in Coxeter, 1987. p. 87).



The fundamental region. Rotation 180° at A2

The diagram illustrates rotation at A2, the midpoint of the fourth square, colour reversed.

The operation is composed so:

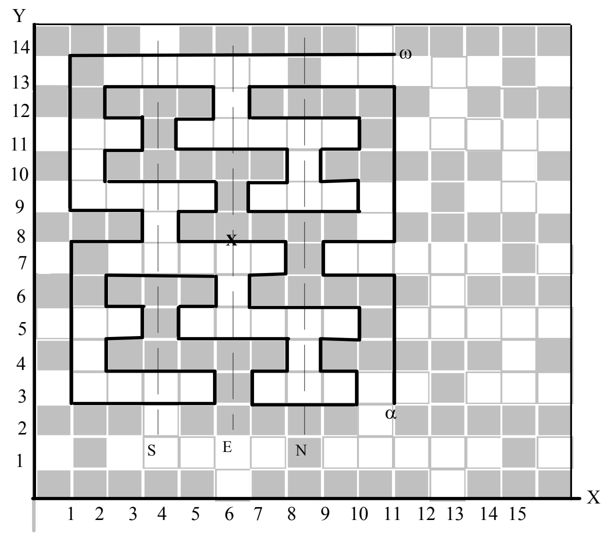
Rotation at A2, colour reversing

L1L0 = rotation 180°(counterclockwise)

L0L1 = rotation 180°(clockwise)

L1L0 = L0L1

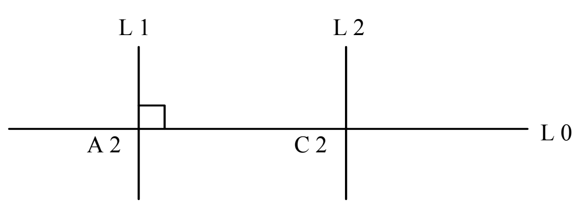
Translation is a movement in a specific direction, (horizontal, vertical, or diagonal) on translation or glide vectors, within clearly defined parameters.



Translations (6,0) and (-2,2) and their opposites (0,6) and (2,-2)
The path between the walls. N, E, S semi-axes indicated.

A translation (6, 0) means each unit is being translated, that is repositioned, six places to the right on the X axis, and zero on the Y axis; (-2, 2) means repositioned negative two on the X axis, and two on the Y axis. The same operations done in reverse are written (0, 6)(2, -2), that is, no movement on the X axis and six on the Y, two on the X, negative two on the Y.

The whole operation is composed so:



180° Rotation with Centre C 2 (Point Reflection)

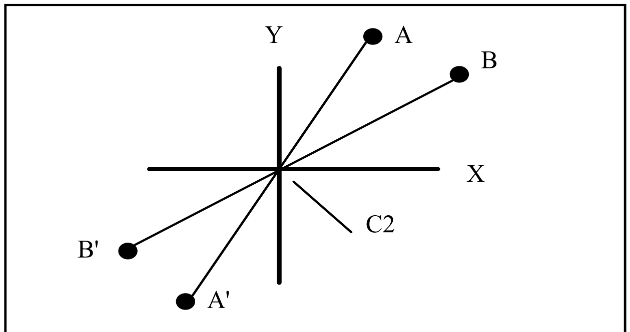
$$
\begin{aligned}
(L\,1\ L\,0)\ (L\,1\ L\,2) &= (L\,0\ L\,1)\ (L1\ L\,2) \\
&= L\,0\ (L\,1\ L\,1)\ L\,2 \quad \text{(associative property)} \\
&= L\,0\ L\,2
\end{aligned}
$$

This equation says:

on the left side perform the operation of 180°counterclockwise rotation on L 1 and L 0, and then a translation or repositioning from L 1 to L 2. This will equal a similar clockwise rotation (L 0 L 1) and translation.

But L 1 and L 1, now associated (an algebraic property), are identical, and cancel each other out, leaving the equality (L 0 L 2), a 180° rotation on point C2.

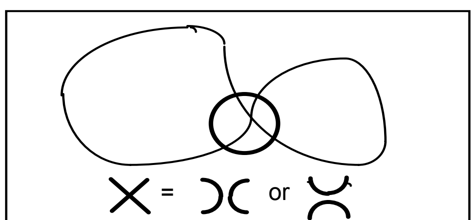
Visually the operation can be represented like this:



180 ° Rotation with centre C2 - Point Reflection

Rosenstiehl calls the resulting pattern the stretched-H array, which generates the turnabouts for the path of the labyrinth. Among the seventeen two-dimensional space groups identified by Federov, that is, the essentially different ways to make "wallpaper" patterns using translations, rotations, reflections and glide-reflections, there are a few which generate all the rest: this is the term Rosenstiehl is using. Crystallographic notation classification P2 is one of these, described as half-turns around a point or vertex, and looks the same either way up (Coxeter, 1987, p. 17).

The turnabouts of the path which come close to each other in opposite directions "evokes the class of problems on touch points and cross points of Jordan Curves."

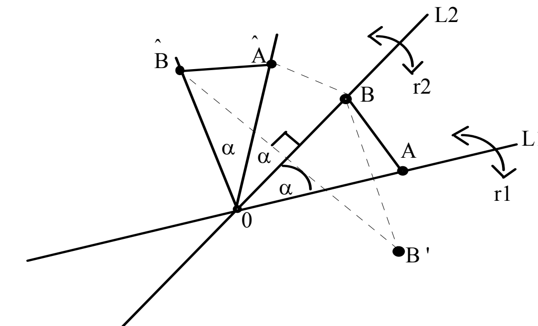


Jordan Curve Crossing Problem

This is related to the parenthesage of lettered corridors mentioned earlier. The crosspoint problem refers to the situation of loops of "words" incident to a junction, marked in the diagram as a circled X. One can choose the direction of the path, allowing the loop to become either a discrete form or two curved lines, depending which alternative you choose, as shown. The evocative quality of this notation is particularly apparent in the Chartres type labyrinth with the curved "labrys" form of turnabout.

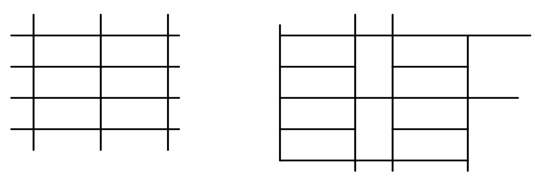
Rosenstiehl describes the sequence changing the pattern of hexominos to birds and fish as "a standard construction [which] transforms one type into the other."

The relationship between the operations of rotation and reflection are illustrated in this diagram:



Isohedral Tiling

Thus the hexominos are reflections and can be considered oppositely congruent. Because the hexominos are the same size and shape throughout, they are monohedral tilings. Because each hexomino is in the same relationship throughout, the tiling is isohedral. "In mathematical terminology, an isohedral tiling is one in which the tiles form one transivity class under the group of symmetries of the tiling, that is, the group of isometries which map the tiling onto itself... Those polygons which permit isohedral tilings have been listed, and the tilings classified: there are 81 types if unmarked, 93 if marked... For isohedral tilings to be of the same “type” it is necessary, but not sufficient, for them to have the same symmetry group, and also for each tile of one tiling to have the same number of adjacents as each tile of the other” (Shephard, G.C. in Coxeter, p. 114).



Monohedral Tilings which are isohedral (left) and not isohedral (right)

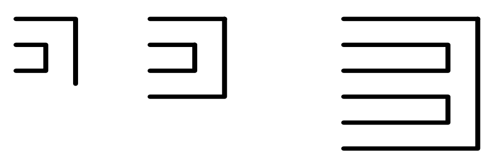
This is not the case with the birds and fish, which are the result of distortion of the grey and white hexominos into two different shapes, so the tiling is dihedral. The fish and birds do not rotate or reflect but rather translate diagonally, preserving their colour; they are classified in Escher's system as BI (rhombus on diagonal glide vectors), as P1 in the crystallographic group system. Of the difficulties in achieving such a transformation, Escher wrote, in his preface to Caroline MacGillavry's *Fantasy and Symmetry* (in Coxeter, 1987 p. 16):

“The border line between two adjacent shapes having a double function, the act of tracing such a line is a complicated business. On either side of it, simultaneously, a recognizability takes shape. But the human eye and mind cannot be busy with two things at the same moment and so there must be a quick and continual jumping from one side to the other.” The version in Rosenstiehl's paper was prepared by his wife, the book illustrator Agnès Rosenstiehl. The transition between stages is made perfectly clear and yet a succesful reproduction requires more than “a little courage” (more skill, anyway, than this writer has); again one wonders, in this collaboration, whose was the greater debt.

Rosenstiehl concludes the presentation with a discussion and theorem of thread mazes. The dodecamaze of three semi-axes and twelve layers (Rosenstiehl uses the centre rosette as a layer in his calculations) is generalised to α semi-axes and 2ℓ layers, with all turnabouts except those on the throat alternating with straight runs on the semi-axes. These mazes he calls (α, 2ℓ)- *alternating mazes*. At the outset he suggests it is unclear whether it is possible for such mazes to exist where $\alpha = 3$ and $\ell = 6$. However his illustrations show one with $\alpha = 7$, $\ell = 8$, and several with $\alpha = 3, \ell = 5$.

The discussion concentrates on the construction of the throat. Because the stretched-H array is the only one generating alternating mazes, several observations can be made. The first is that the size of the piece cut off in the array, that is, what remains after the removal of the frame, defines the parameters α and ℓ. The axes can be followed through the morph from their original radiant siting to the positions they occupy parallel to one another and perpendicular to the layers that represent the concentric circuits. In the diagram they are marked N, E, S in accordance with their original location on the circular form. Because the stretched-H carries the path across the axes along the lengths of the “upright” elements of the H-shape, alternation of straight runs and turnabouts is inevitable. Similarly, the number of circuits is dependent upon the boundary separating the piece from the infinite repeat (a courageous person with a graphics program could carry out a morph of the ($\alpha = 7$, $\ell = 8$) example to verify this). Together these choices in height and width determine the range and domain of the path.

The second observation concerns location of the frame in determining the type of nesting configuration in the throat: Rosenstiehl calls this the vertical cuts in the array. Two types of configuration are possible, Type F (for FOUR) has one turnabout pair nested inside a second, wider one; type S (for SIX), has two turnabout pairs nested beside each other inside a third, wider one. At most, he says, there can be two Fs, and an “arbitrary” number of S configurations placed consecutively and separating the Fs if there are two. This can look a little confusing in a graphic depiction, because Rosenstiehl includes α and ω as elements for calculation purposes:



Nested configurations of the throat, F and S

Rosenstiehl's final observation concerns translation properties: the left and right cuts define identical or symmetrical systems of nested pairs. Identical would mean the throat halves were reflections of each other; that is, the products of an opposite isometry, whereas the quality of being symmetrical involves the direct isometry operations of translation or rotation. As we have seen, both these isometries are engaged when generating the array from the hexomino, so it follows that the pattern also would reveal these qualities as "isohedrality is a global condition of tiling" (Shephard, p. 114). This is apparent in the diagram of the path through the field: eliminating ω , looking only at colour reversal, and bearing in mind that "a perfect two-colour tiling has the property that a symmetry of the tiling either keeps all the tiles their original colour, or changes all the tiles to the opposite colour" (Schattscheider, p. 83) we find the rotation point on the central "East" axis on the line between the sixth and seventh circuits (marked X on the diagram). Rotation here reverses the colour as well as the relative position of the turnabout which forms the outer arms of the S configurations dividing the tiers of the throat.

Rosenstiehl takes into account the difference in results from reflection and rotation in his first theorem of $(\alpha, 2\ell)$ - alternating mazes:

In all $(\alpha, 2\ell)$ - alternating mazes the left and right nested systems of the throat are identical or symmetrical and belong to six possible types: (F), (F, F), (F, S..., S, F) (F, S, ..., S), (S, ..., S, F), or (S, ..., S).

It follows that the number of layers is partitioned as:

$$2\ell = 4\alpha + 6k + 4\beta \text{ with } \alpha, \beta = 0 \text{ or } 1, k \geq 0$$

where α and β are F indicators and k is the number of S configurations in the system. The dodecamaze can be obtained only by the solution $\alpha = \beta = 0$, k = 2, which means that its throat systems are necessarily of the type (S, S)."

The mechanics of an investigation similar to this were described in *Dodécadédale*, where the five possible schematics of parenthesage were laid out. But at that point Rosenstiehl was still relying on both the law of alternation and the law of doubling, and he has stated in *The Path* that he is working only with alternation at this stage. We can take "partitioning" of the number of layers to refer to the two-tiered construction, circuits one through six and seven through twelve (remembering that the centre is included as a circuit). It is clear he is referring in this equation to one side of the throat:

$$2\ell = 4\alpha + 6k + 4\beta$$

$$12 = (4x0) + (6x2) + (4x0)$$

$$12 = 12$$

and one can see from the drawings and diagrams that there are indeed two S systems on each side of the throat. The "mathematical fantasy" in his appendix showing $(a = 7, \ell = 8)$ is type (F, S, S) and works out as:

$$2\ell = 4\alpha + 6k + 4\beta$$

$$16 = (4x1) + (6x2) + (4x0)$$

$$16 = 4 + 12 + 0$$

The second theorem states:

The dodecamaze is the only (3, 12) - alternating maze. Given the conditions α, β = 0 or 1, k $\geq$ 0, no alternative is possible.

The third theorem states:

There exist (α,2ℓ) - alternating mazes only for a =3h or 3h-2, with h > 0. When 2ℓ has a unique partition (2ℓ = 4 α + 6 k + 4 β; *with* α, β = 0 *or* 1, k $\geq$ 0) *it is necessary that α = 3h, and the maze is unique. When* 2 l *has two distinct partitions* (*corresponding to* α = 1, β = 0*, and* α = 0, β = *1) each one generates a maze, with its two throat systems identical if α = 3h, and symmetrical if α = 3h - 2.*

This theorem is illustrated by the (2ℓ = 16) "mathematical fantasy" maze described above. The condition (α = 3h - 2) is met with h = 3 giving α = 7. The throat systems are (F, S, S) and (S, S, F), reversed by rotation, an opposite symmetry operation. The condition of "two distinct partitions" seems to suggest that α and β are found distributed one to each side of a throat system, though whether that is all they indicate, or whether a system can contain both on one side, is not obvious.

Citations

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Note: Tessa Morrison has used Rosenstiehl's material to develop classifications of "simple, alternating, transitive" unicursal labyrinths according to their topology, nuclei, fundamental forms and elements. See: "The Geometry of History; 032147658." *Visual Mathematics*. Vol. 3, No 4, 2001. http://www.mi.sanu.ac.yu/vismath/morrison/index.html. Accessed May 14, 2006

“Escherisation” of the eleven circuit cathedral labyrinth in three stages. Rosenstiehl (1987)

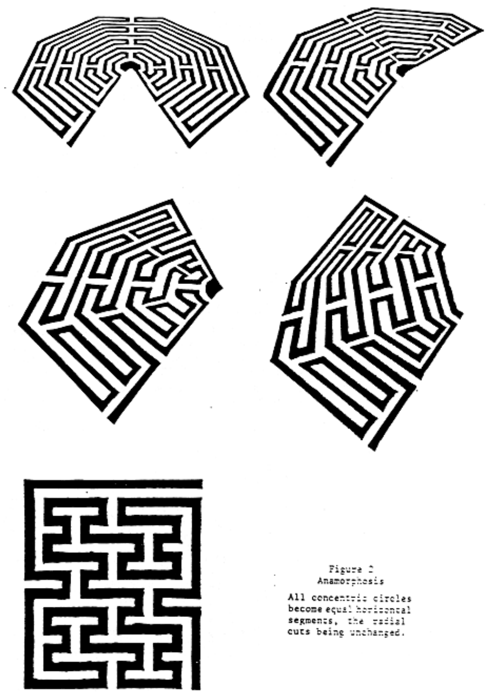

Figure 2
Anamorphosis
All concentric circles become equal horizontal segments, the radial cuts being unchanged.

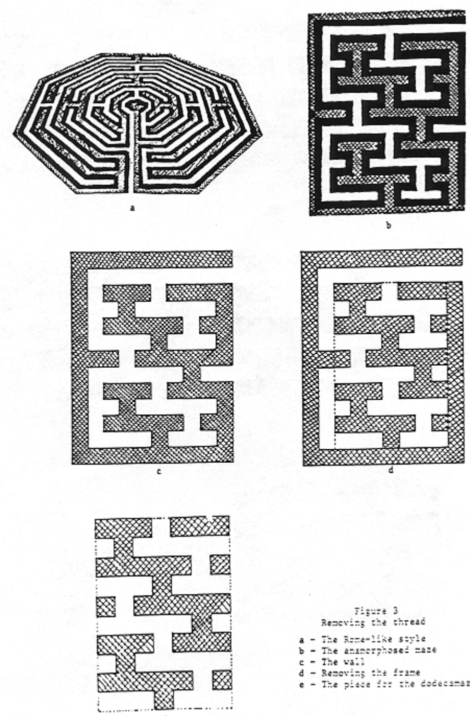

Figure 3
Removing the thread

a - The Rome-like style
b - The anamorphosed maze
c - The wall
d - Removing the frame
e - The piece for the dodecamaze

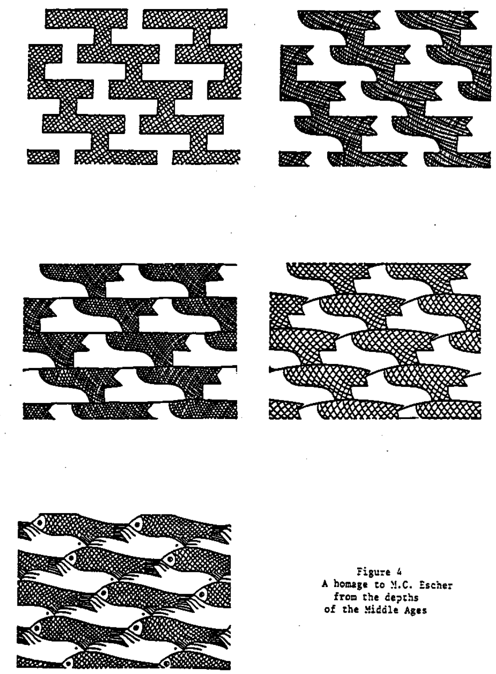

Figure 4
A homage to M.C. Escher
from the depths
of the Middle Ages